

Vortex States Induced by Proximity Effect in Hybrid Ferromagnet-Superconductor Systems

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We consider superconductivity nucleation in multiply connected mesoscopic samples such as thin-walled cylinders or rings placed in electrical contact with a ferromagnet. The superconducting critical temperature and order parameter structure are studied on the basis of linearized Usadel equations. We suggest a mechanism of switching between the superconducting states with different vorticities caused by the exchange field and associated with the oscillatory behavior of the Cooper pair wave function in a ferromagnet.

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I. INTRODUCTION

The origin of the vortex states in hybrid ferromagnet (F)-superconductor (S) structures is closely related to the basic mechanisms responsible for the interplay between the ferromagnetic and superconducting orderings (see, e.g., Ref.¹). The first mechanism is associated with the orbital effect, i.e., interaction of Cooper pairs with the magnetic field induced by magnetic moments². This field causes the appearance of inhomogeneous superconducting phase distributions and spontaneous vortex states. The switching between these states characterized by different winding numbers can result in an oscillatory behavior of the critical temperature T_c as a function of the external field H , which resembles the Little-Parks effect in multiply connected superconducting samples³. Such nonmonotonic behavior of $T_c(H)$ was shown to be inherent to hybrid F/S systems with magnetic dots or domains^{4,5,6}, that create a "magnetic template" for nucleation of superconducting order parameter. The second mechanism arises from the exchange interaction which comes into play because of the proximity effect, when the Cooper pairs penetrate into the F layer and induce superconductivity there. The latter effect is known to result in the damped-oscillatory behavior of the Cooper pair wave function in a ferromagnet¹ which is the cause of a number of fascinating interference phenomena in hybrid F/S structures. In particular, this peculiar proximity effect reveals itself in the oscillating^{7,8} or re-entrant⁹ behavior of the critical temperature as a function of a ferromagnetic layer thickness in layered F/S structures, and is responsible for the formation of π -junctions^{10,11,12}. A Josephson π -junction is a generic example of the system where the proximity effect in a ferromagnetic subsystem is used to obtain an energetically favorable superconducting state with a nontrivial distribution of the order parameter phase. A resulting distinctive feature of the systems with π -junctions is a possible unusual ground state with spontaneous supercurrents and, in particular, with spontaneously formed vortices. For a Josephson junction with a step-like change in the F layer thickness such

spontaneously formed vortex states have been discussed, e.g., in Refs.^{13,14,15}.

It is the purpose of this paper to examine a possibility to realize a switching between the spontaneously created vortex states in multiply connected samples caused by the proximity effect with a ferromagnet. We focus on the behavior of critical temperatures for superconducting states with different vorticities and, thus, in some sense study an analog of the Little-Parks effect caused by the exchange interaction mechanism.

To elucidate our main results we start from a qualitative discussion of the proximity effect on the superconducting ground state in a thin-walled superconducting shell surrounding a cylinder (core) of a ferromagnetic metal (see Fig. 1). We expect that the superconducting ground state in such geometry should be strongly influenced by the damped-oscillatory behavior of the superconducting order parameter in ferromagnetic cylinder and, thus, controlled by the ratio of the period of the order parameter oscillations ($\sim \xi_f$) to the radius R_f of the F core. Indeed, for $R_f < \xi_f$, the variation of the pair

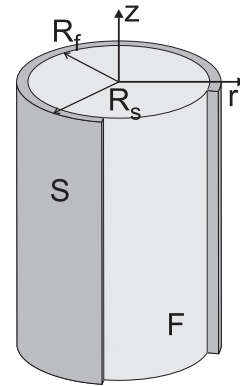


FIG. 1: The schematic representation of the F/S system under consideration: thin-walled superconducting shell around a ferromagnetic cylinder. Here R_f is the radius of the F core, and R_s is the outer radius of the S shell, (r, θ, z) is the cylindrical coordinate system.

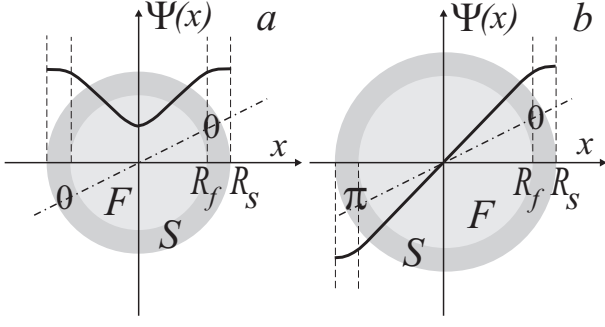


FIG. 2: The schematic behavior of the superconducting order parameter inside the F cylinder. (a) The curve $\Psi(x)$ represents sketchy the behavior of the pair wave function in the $L = 0$ phase. Due to symmetry the derivative $\partial_x \Psi$ is zero at the center of F cylinder. (b) The pair wave function in the phase with $L = 1$ vanishes at the center of F cylinder, and $\Psi(x)$ has a π -shift in diametrically opposite points.

wave function Ψ along a line crossing the F core appears to be modest and the order parameter can not change its sign along the line. It means that in the ground state the superconducting phase in diametrically opposite points must be the same. The resulting angular momentum L of the pair wave function Ψ is equal to zero (see Fig. 2a). This state is analogous to the 0-phase state of SFS layered structures¹¹. For a larger radius $R_f \gtrsim \xi_f$, the pair wave function may cross zero at the axis of the F cylinder which causes a π -shift in the superconducting phase in diametrically opposite points. In this case the angular momentum L of the pair wave function Ψ is nonzero (Fig. 2b). Thus, the penetration of Cooper pairs into the F core and exchange interaction can induce the superconducting states with a nonzero vorticity. It is natural to expect that the supercurrents flowing in such states with $L \neq 0$ are responsible for an additional contribution to the vortex energy. The interplay between the exchange effect and supercurrent depending energy term may result in a subsequent switching between the states with different vorticities, as the F core radius increases. An obvious consequence of these transitions between the states with different L should be a nonmonotonic dependence of the critical temperature T_c on the F core radius and exchange field.

The paper is organized as follows. In Sec. II we briefly discuss the basic equations. In Sec. III we study the switching between different vortex states in two model F/S systems. The first system consists of a thin-walled superconducting cylindrical shell surrounding a cylinder of a ferromagnetic metal. The second one is a planar structure which consists of a mesoscopic superconducting ring placed at the surface of a thin ferromagnetic film. For both cases we assume that there is a good electrical contact between the F and S regions, to assure a rather strong proximity effect. We summarize our results in Sec. IV.

II. MODEL

We assume the elastic electron-scattering time τ to be rather small, so that the critical temperature T_c and exchange field h satisfy the dirty-limit conditions $T_c \tau \ll 1$ and $h \tau \ll 1$. In this case a most natural approach to calculate T_c is based on the Usadel equations¹⁶ for the averaged anomalous Green's function F_f and F_s for the F and S regions, respectively (see¹ for details). Near the second-order superconducting phase transition, the Usadel equations can be linearized with respect to the pair potential $\Delta(\mathbf{r})$. In the F (S) region these linearized Usadel equations take the form

$$-\frac{D_f}{2} \nabla^2 F_f + (|\omega| + i h \operatorname{sgn} \omega) F_f = 0, \quad (1)$$

$$-\frac{D_s}{2} \nabla^2 F_s + |\omega| F_s = \Delta(\mathbf{r}). \quad (2)$$

The superconducting critical temperature T_c is determined from the self-consistency condition for the gap function:

$$\Delta(\mathbf{r}) \ln \frac{T_c}{T_{c0}} + \pi T_c \sum_{\omega} \left(\frac{\Delta(\mathbf{r})}{|\omega|} - F_s(\mathbf{r}, \omega) \right) = 0. \quad (3)$$

Here D_f and D_s are the diffusion constants in the ferromagnet and superconductor, respectively, and $\omega = (2n+1)\pi T_c$ is a Matsubara frequency at the temperature T_c . Equations (1),(2) must be supplemented with the boundary condition at the outer surfaces

$$\partial_{\mathbf{n}} F_{f,s} = 0, \quad (4)$$

and at the interface between the F and S metals:¹⁷

$$\sigma_s \partial_{\mathbf{n}} F_s = \sigma_f \partial_{\mathbf{n}} F_f; \quad F_s = F_f + \gamma_b \xi_s \partial_{\mathbf{n}} F_f. \quad (5)$$

Here $\xi_s = \sqrt{D_s/2\pi T_{c0}}$ is the superconducting coherence length, σ_f and σ_s are the normal-state conductivities of the F and S metals, γ_b is related to the F/S boundary resistance R_b per unit area through $\gamma_b \xi_s = R_b \sigma_f$, and $\partial_{\mathbf{n}}$ denotes a derivative taken in the direction perpendicular to the outer surfaces or to the F/S interface. For the sake of simplicity we assume $h \gg \pi T_{c0}$ and neglect the proximity effect suppression caused by a finite F/S interface resistance¹⁸, i.e., take $\gamma_b \rightarrow 0$. In this regime we get $F_f = F_s$ at the F/S interface.

For a system with a cylindrical symmetry the vorticity parameter L just coincides with the angular momentum of the Cooper pair wave function. Choosing cylindrical coordinates (r, θ, z) we look for solutions of the equations (1),(2),(3) characterized by certain angular momenta L :

$$\Delta(\mathbf{r}) = \Delta(r, z) e^{iL\theta}, \quad F_{f,s}(\mathbf{r}) = f_{f,s}(r, z) e^{iL\theta}, \quad (6)$$

According to the equations (1),(2),(3) there is a symmetry $F_{f,s}(\omega) = F_{f,s}^*(-\omega)$, so that we can treat only positive

ω values. The Usadel equations (1),(2) can be written in the form

$$-\frac{D_f}{2} \left(\frac{1}{r} \partial_r (r \partial_r f_f) + \partial_z^2 f_f - \frac{L^2}{r^2} f_f \right) + i \hbar f_f = 0, \quad (7)$$

$$-\frac{D_s}{2} \left(\frac{1}{r} \partial_r (r \partial_r f_s) + \partial_z^2 f_s - \frac{L^2}{r^2} f_s \right) + \omega f_s = \Delta. \quad (8)$$

An appropriate self-consistency equation (3) can be rewritten as follows:

$$\Delta \ln \frac{T_c}{T_{c0}} + 2\pi T_c \sum_{\omega > 0} \left(\frac{\Delta}{\omega} - \text{Re } f_s(\omega) \right) = 0. \quad (9)$$

III. CRITICAL TEMPERATURE OF VORTEX STATES

Now we proceed with the critical temperature calculations for different vortex states. For the sake of definiteness we consider here two generic examples of hybrid F/S systems which we believe to manifest the vorticity switching scenario suggested in the introduction.

A. Thin-walled superconducting shell around a ferromagnetic cylinder

Consider a superconducting cylindrical shell of a thickness $W = R_s - R_f \ll R_f$ surrounding a thin cylinder of a ferromagnetic metal with a uniform magnetization $\mathbf{M} = M\mathbf{z}_0$. Here R_f is the radius of the F core, and R_s is the outer radius of the S shell (see Fig. 1). Naturally, to observe the pronounced influence of the proximity effect on the transition temperature, the thickness of the S shell W must be smaller than the superconducting coherence length ξ_s .

For a thin and long F cylinder with the magnetization direction chosen along the z -axis the magnetic field B vanishes outside the F region. However, this magnetization induces a vector potential $A_\theta \simeq 2\pi M R_f$ in the S region, which may result in a standard Little-Parks effect of electromagnetic origin. We may take account of this vector potential in Eq. (8) replacing the vorticity parameter L by the value $L - \Phi/\Phi_0$, where $\Phi_0 = 2\pi\hbar c/e$ is the magnetic flux quantum and $\Phi = 4\pi^2 M R_f^2$ is the total magnetic flux. For the standard Little-Parks effect the critical temperature vs Φ oscillates with a period Φ_0 and an amplitude $\Delta T_c \sim T_c \xi_s^2/R_f^2$. For typical parameters $M \sim 10^2$ G, $T \sim 10$ K, $D_s \sim 10$ cm²/s and R_f of order of several $\xi_f \sim 10$ nm lengths we get $\Phi/\Phi_0 \ll 1$ and $\Delta T_c \ll T_c$. These simple estimates allow us to find a region of parameters where we can exclude the effect of magnetic field on T_c . The above effect of the magnetic field can be also weakened provided we decrease the height of the F/S cylinder going over to the case of a thin

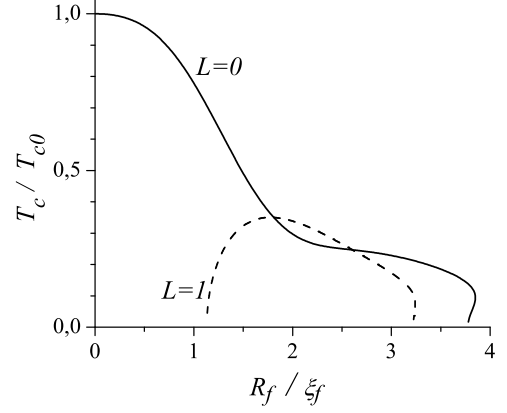


FIG. 3: The dependence of the critical temperature T_c on the F core radius R_f for two values of the vorticity $L = 0$ (solid line) and $L = 1$ (dashed line). Here we choose $W = 0.5\xi_s$; $\sigma_s/\sigma_f = 2.5$; $\xi_s/\xi_f = 0.265$.

disk when the field B is suppressed due to the demagnetization factor. Note that choosing the magnetization direction in the plane perpendicular to the cylinder axis we can get rid of the standard Little-Parks effect completely because of the absence of the magnetic field component along the cylinder axis. On the contrary, the vorticity switching scenario studied below does not depend on the magnetization direction.

We look for homogeneous along z solutions of the equations (7), (8) with a certain angular momentum L . In this case the equation (7) in the F cylinder can be readily solved:

$$f_f = C I_L(q_f r), \quad q_f = \frac{1+i}{\xi_f}. \quad (10)$$

Here $I_L(u)$ is the modified Bessel function of first kind of order L , and $\xi_f = \sqrt{D_f/\hbar}$ is the characteristic length scale of the order parameter variation in the F metal. In the dirty limit parameter ξ_f determines both the length scale of oscillations and decay length for the Cooper pair wave function in a ferromagnet¹. The boundary conditions (4),(5) for Eq. (8) take the form:

$$\sigma_s \left. \frac{df_s}{dr} \right|_{R_f} = \alpha_L q_f \sigma_f f_s(R_f), \quad \left. \frac{df_s}{dr} \right|_{R_s} = 0, \quad (11)$$

$$\alpha_L = \frac{L}{u_f} + \frac{I_{L+1}(u_f)}{I_L(u_f)}, \quad u_f = q_f R_f.$$

For $W \ll \xi_s$, the variations of the functions $f_s(r)$ and $\Delta(r)$ in the superconducting shell are small: $f_s(r) \simeq f$, $\Delta(r) \simeq \Delta$. Therefore, we can average Eq. (8) over the thickness of the S shell, using the boundary conditions (11) to integrate the term $\partial_r(r \partial_r f_s)$. Finally, we obtain

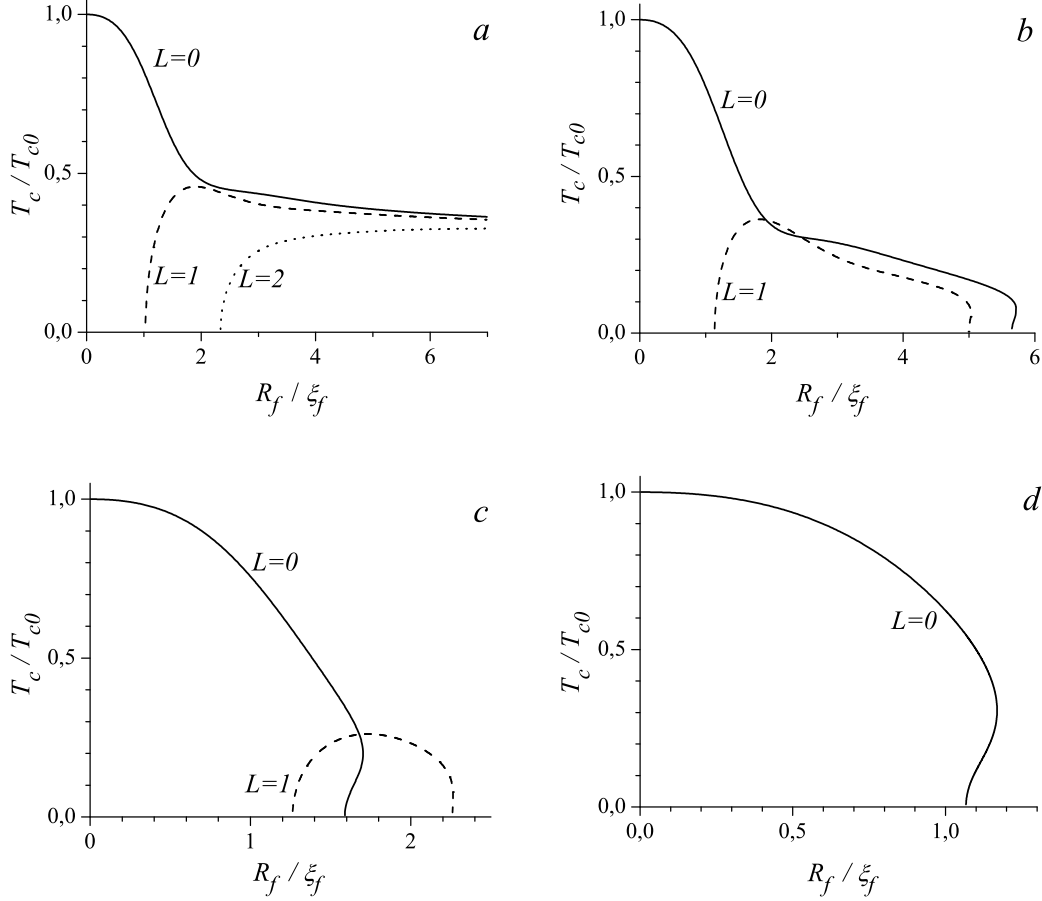


FIG. 4: The typical dependences of the critical temperature T_c on the F core radius R_f for different values of the vorticity $L = 0$ (solid line), $L = 1$ (dashed line) and $L = 2$ (dotted line). Here we choose $W = 0.5\xi_s$; $\xi_s/\xi_f = 0.28$ and different values of the ratio σ_s/σ_f : a) $\sigma_s/\sigma_f = 3$; b) $\sigma_s/\sigma_f = 2.7$; c) $\sigma_s/\sigma_f = 2.5$; d) $\sigma_s/\sigma_f = 2$.

the following expression:

$$f = \frac{\Delta}{\omega + \frac{D_s}{2} \left[\left(\frac{L}{R_f} \right)^2 + \frac{\sigma_f q_f}{\sigma_s W} \alpha_L \right]}. \quad (12)$$

Substituting Eq.(12) into Eq.(9) one obtains a self-consistency equation for the critical temperature T_c :

$$\ln \frac{T_c}{T_{c0}} = \Psi \left(\frac{1}{2} \right) - Re \Psi \left(\frac{1}{2} + \Omega_L \right), \quad (13)$$

where Ψ is the digamma function. The depairing parameter

$$\Omega_L = \frac{1}{2} \frac{T_{c0}}{T_c} \xi_s^2 \left[\left(\frac{L}{R_f} \right)^2 + \frac{\sigma_f q_f}{\sigma_s W} \alpha_L \right] \quad (14)$$

is responsible for the superconductivity destruction in the shell due to both the exchange effect and the supercurrent flowing around the cylinder.

Figure 3 shows a typical dependency of the critical temperature T_c on the F core radius R_f , obtained from Eqs. (13),(14) for different winding numbers L . We see that for a small F cylinder radius $R_f \ll \xi_f$ only the state with $L = 0$ appears to be energetically favorable. The influence of the proximity effect is weak and the critical temperature T_c is close to T_{c0} . The T_c of a vortex state with $L \neq 0$ is suppressed because of a large supercurrent energy. The increase in the radius R_f results in a decrease in T_c for the state with $L = 0$ and reduce the kinetic energy of supercurrents for $L \neq 0$. At the same time, the damped-oscillatory behavior of the superconducting order parameter in a ferromagnet becomes important. If the diameter of the F cylinder is comparable with the period of the order parameter oscillations ($\sim \xi_f$) and satisfies approximately the conditions of the π -phase superconductivity in layered F/S structures ($2\xi_f < 2R_f < 5\xi_f$)¹, then there appears a π -shift in the phase of the superconducting order parameter in diametrically opposite points (see Fig. 2b). In this case, the critical temperature of $L = 1$ state becomes higher

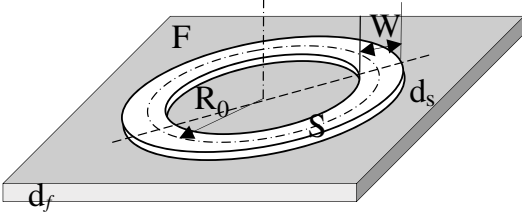


FIG. 5: Superconducting ring lying at the surface of a thin ferromagnetic film. Here R_0 and W are the radius and the width of the S ring, and d_f (d_s) is the thickness of a ferromagnetic (superconducting) layer.

than the critical temperature of the state with $L = 0$. Thus, our calculations confirm the qualitative arguments given in the introduction. The penetration of Cooper pairs into the F core and the phase shift of the pair wave function Ψ due to the exchange interaction can induce vortex states in the superconducting shell.

To illustrate the scenario of switching between the states with different vorticities L and a nonmonotonic dependence of the critical temperature T_c vs R_f we present here several $T_c(R_f)$ curves for various conductivity ratio (see Fig. 4). We see that the ratio σ_s/σ_f of the normal-state conductivities of the F and S metals is an important factor, controlling the generation of the vortex states in the F/S structure under consideration.

B. Superconducting ring at the surface of a thin ferromagnetic film

As a second example we consider a planar hybrid system, i.e., a superconducting ring lying at a ferromagnetic film with a uniform in-plane magnetization \mathbf{M} (see Fig. 5). Such version of the setup can be more convenient for the experimental observation of the switching phenomena. The superconducting ring of the radius R_0 and the width $W \ll R_0$ occupies the region $R_0 - W/2 < r < R_0 + W/2$, $0 < z < d_s$. The boundary conditions at the interfaces with vacuum yield $\partial_r f_s(R_0 \pm W/2, z) = 0$, and $\partial_z f_s(r, d_s) = 0$. For simplicity, we consider only the case of a rather thin ring with $W \ll \xi_{f,s}$ and $d_s \ll \xi_s$ which allows us to assume the variations of the functions $f_s(r)$ and $\Delta(r)$ in the superconducting ring to be small. Thus, we can average Eq. (8) over the volume of the S ring, integrate the terms $\partial_r(r \partial_r f_s)$ and $\partial_z^2 f_s$, and make use of the boundary condition at the interface with vacuum. Finally, we get the following expression for the derivative $\partial_z f_s$ at $r = R_0$, $z = 0$:

$$\frac{1}{d_s} \partial_z f_s = \frac{2}{D_s} (\Delta - \omega f_s) - \left(\frac{L}{R_0} \right)^2 f_s. \quad (15)$$

The F metal occupies the region $-d_f < z < 0$. We will address the case of a very thin F film: $d_f \ll \xi_f$. The boundary conditions at the interfaces with vacuum yield $\partial_z f_f = 0$. Far from the superconducting ring, i.e. for $r - R_0 - W/2 \gg d_f$ and for $R_0 - W/2 - r \gg d_f$ we can average the Usadel equation (7) over the thickness d_f :

$$-\frac{D_f}{2} \left(\frac{1}{r} \partial_r(r \partial_r f_f) - \frac{L^2}{r^2} f_f \right) + \imath h f_f = 0. \quad (16)$$

The solution of this equation reads:

$$f_f(r) = \begin{cases} C_1 I_L(q_f r) & , R_0 - W/2 - r \gg d_f \\ C_2 K_L(q_f r) & , r - R_0 - W/2 \gg d_f \end{cases} \quad (17)$$

where $I_L(u)$ and $K_L(u)$ are the modified Bessel functions of order L . Making use of this solution one can easily get the following relations between the function f_f and its derivative $\partial_r f_f$ at $r = R_0 \pm \varepsilon$:

$$\left. \frac{df_f}{dr} \right|_{R_0 - \varepsilon} = q_f \left(\frac{L}{u_0} + \frac{I_{L+1}(u_0)}{I_L(u_0)} \right) f_f(R_0 - \varepsilon), \quad (18)$$

$$\left. \frac{df_f}{dr} \right|_{R_0 + \varepsilon} = q_f \left(\frac{L}{u_0} - \frac{K_{L+1}(u_0)}{K_L(u_0)} \right) f_f(R_0 + \varepsilon), \quad (19)$$

where $u_0 = q_f R_0$ and $\max[W, d_f] \ll \varepsilon \ll \xi_f \lesssim R_0$. Assuming f_f to be a slow function of r one can write a boundary condition on the derivative jump:

$$\left. \frac{df_f}{dr} \right|_{R_0 - \varepsilon}^{R_0 + \varepsilon} \simeq -q_f Q_L f_f(R_0), \quad (20)$$

$$Q_L = \frac{I_{L+1}(u_0)}{I_L(u_0)} + \frac{K_{L+1}(u_0)}{K_L(u_0)}.$$

On the other hand in the region $|r - R_0| \ll \xi_f$ the Eq. (7) takes a simple form:

$$\frac{1}{r} \partial_r(r \partial_r f_f) + \partial_z^2 f_f = 0.$$

Integrating this equation over the region $R_0 - \varepsilon < r < R_0 + \varepsilon$ and over the ferromagnetic film thickness and making use of the boundary conditions described above we obtain:

$$W \partial_z f_f \Big|_{z=0, r=R_0} = q_f d_f Q_L f_f(R_0) \quad (21)$$

As before we restrict ourselves to the case of low F/S interface resistance assuming $\gamma_b = 0$ in (5). In this regime, $f_f(R_0, 0) \simeq f_s(R_0, 0) \equiv f$. The Eqs. (15), (21) and the boundary conditions at the F/S interface (5) determine the amplitude f :

$$f = \frac{\Delta}{\omega + \frac{D_s}{2} \left[\left(\frac{L}{R_0} \right)^2 + \frac{q_f}{\eta W} Q_L \right]}, \quad (22)$$

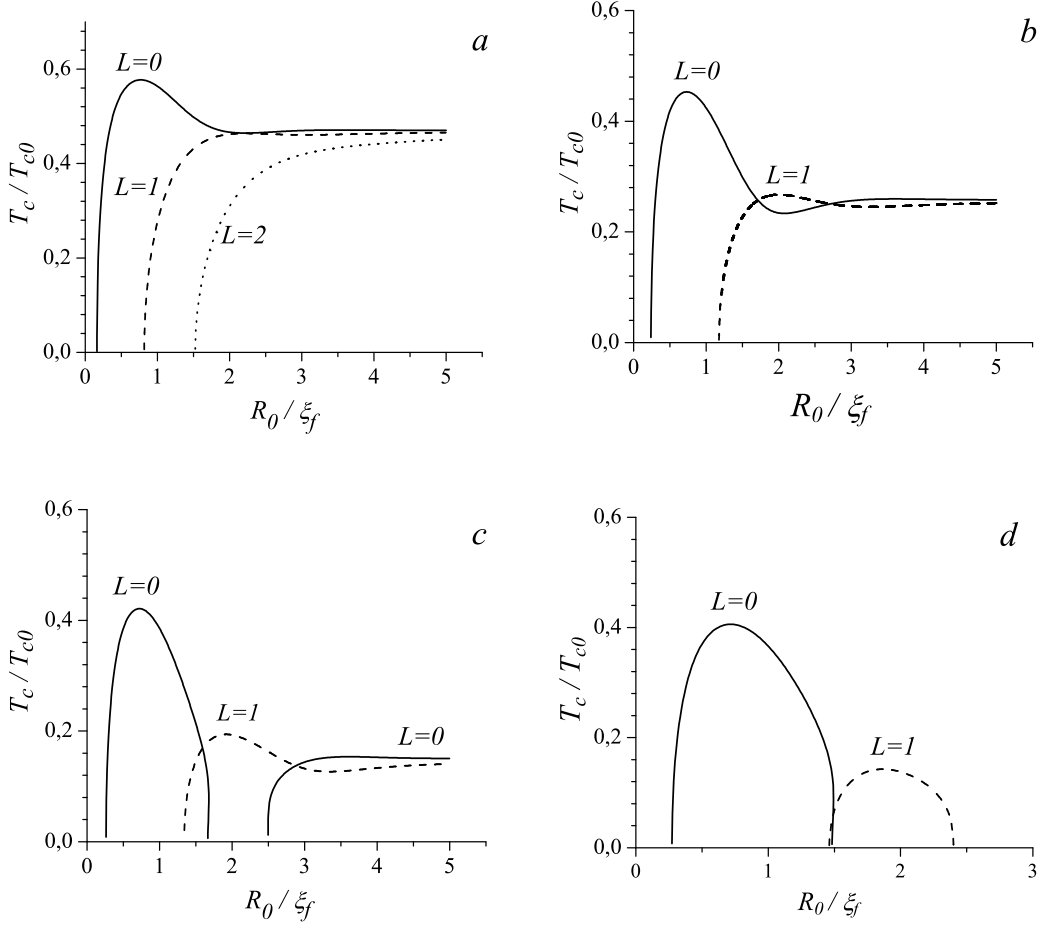


FIG. 6: The dependence of the critical temperature T_c on the S ring radius R_0 for different values of the vorticity $L = 0$ (solid line), $L = 1$ (dashed line) and $L = 2$ (dotted line). Here we choose $d_s/d_f = 1$, $W = 0.5\xi_s$, $\xi_s/\xi_f = 0.1$: a) $\sigma_s/\sigma_f = 2.5$; b) $\sigma_s/\sigma_f = 2.1$; c) $\sigma_s/\sigma_f = 2.03$; d) $\sigma_s/\sigma_f = 2.0$.

where $\eta = \sigma_s d_s / \sigma_f d_f$. Substitution of (22) into Eq.(9) results in the self-consistency equation for the critical temperature T_c of the F/S hybrid (13), where the depairing parameter Ω_L is determined by the following expression:

$$\Omega_L = \frac{1}{2} \frac{T_{c0}}{T_c} \xi_s^2 \left[\left(\frac{L}{R_0} \right)^2 + \frac{q_f}{W\eta} Q_L(u_0) \right]. \quad (23)$$

In Fig. 6, we present typical dependences of the critical temperature T_c on the S ring radius R_0 for the different orbital number L , obtained from Eqs. (13), (23). The curves appear to be qualitatively similar to the ones obtained for a superconducting thin-walled cylinder in the previous subsection. Note that for a particular choice of parameters (see Fig.6c) T_c vanishes in a certain interval of R_0 values and we observe an interesting re-entrant behavior of the critical temperature for a state with zero vorticity.

IV. SUMMARY

To summarize, we suggest a mechanism of switching between the superconducting states with different vorticities caused by the exchange field in the hybrid S/F structures and associated with the oscillatory behavior of the Cooper pair wave function in a ferromagnet. We defined the range of the system parameters at which the predicted effect can be experimentally observable. The most restrictive condition is imposed on the relation between the superconducting cylinder or ring radius R_0 and coherence lengths ξ_s and ξ_f . On the one hand the radius should essentially exceed the superconducting coherence length to decrease the kinetic energy of supercurrents in the states with nonzero vorticity, but on the other hand it should be of the order of only several ξ_f lengths to ensure a rather strong influence of the proximity effect. As a result, to observe the switching effect we need to consider the S/F systems with a rather large ratio ξ_f/ξ_s . This ratio can be increased if we choose a superconduct-

ing material with a rather short coherence length, e.g., heavy fermion compounds¹⁹. Despite of such restriction we believe that the proximity induced switching between the vortex states can be experimentally observable and would provide an interesting manifestation of interference effects in mesoscopic superconductivity.

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